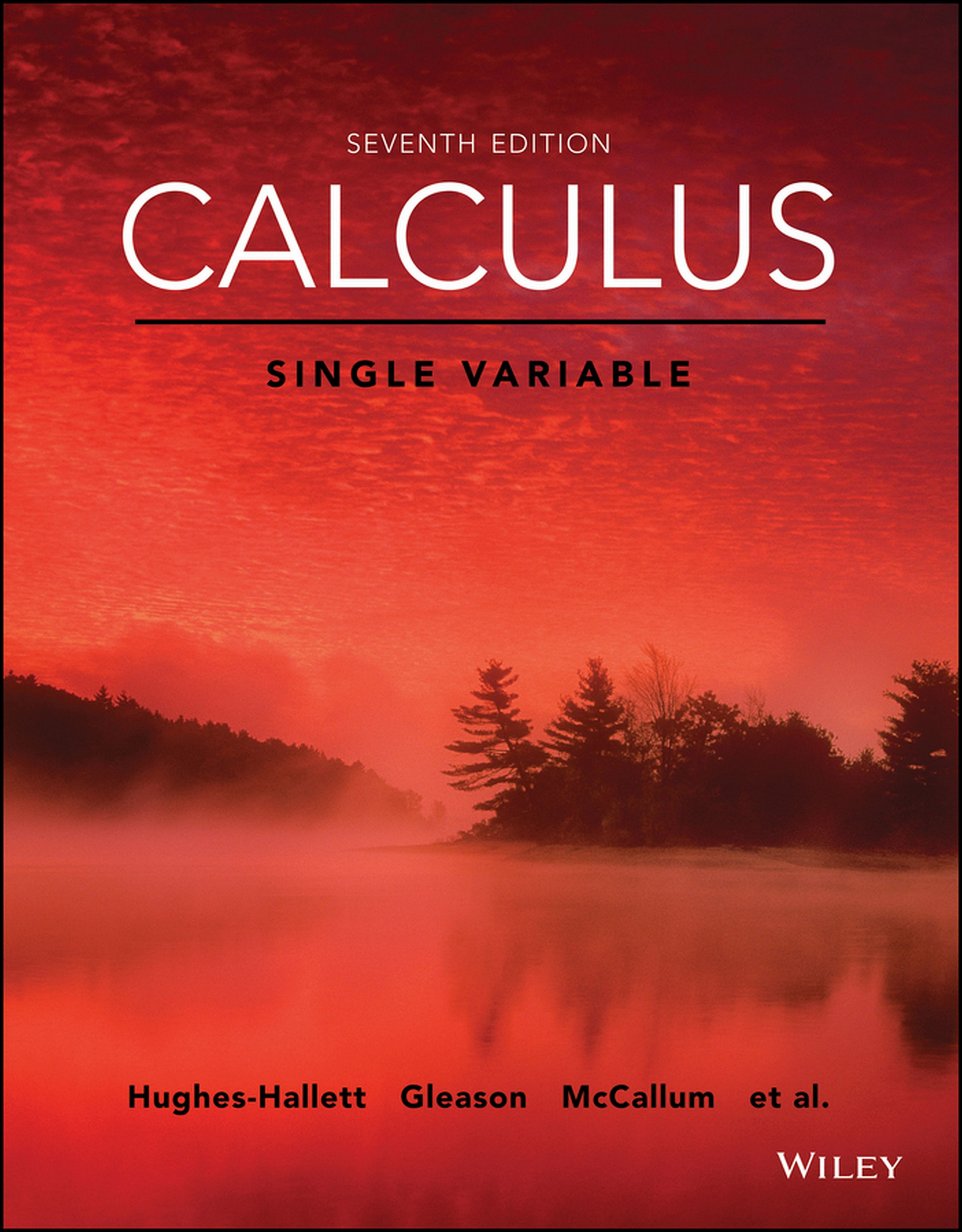


SEVENTH EDITION

CALCULUS

SINGLE VARIABLE

Hughes-Hallett Gleason McCallum et al.

WILEY

Lines

Slope of line through (x_1, y_1) and (x_2, y_2):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-slope equation of line through (x_1, y_1) with slope m:

$$y - y_1 = m(x - x_1)$$

Slope-intercept equation of line with slope m and y-intercept b:

$$y = b + mx$$

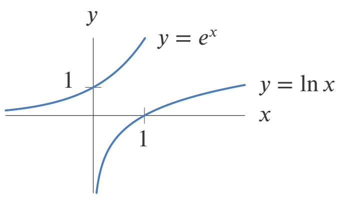
Rules of Exponents

$$a^x a^t = a^{x+t}$$
$$\frac{a^x}{a^t} = a^{x-t}$$
$$(a^x)^t = a^{xt}$$

Definition of Natural Log

$y = \ln x$ means $e^y = x$

ex: $\ln 1 = 0$ since $e^0 = 1$



Identities

$$\ln e^x = x$$
$$e^{\ln x} = x$$

Rules of Natural Logarithms

$$\ln(AB) = \ln A + \ln B$$
$$\ln\left(\frac{A}{B}\right) = \ln A - \ln B$$
$$\ln A^p = p \ln A$$

Distance and Midpoint Formulas

Distance D between (x_1, y_1) and (x_2, y_2):

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of (x_1, y_1) and (x_2, y_2):

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Quadratic Formula

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Factoring Special Polynomials

$$x^2 - y^2 = (x + y)(x - y)$$
$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$
$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

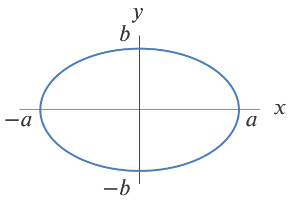
Circles

Center (h, k) and radius r:

$$(x - h)^2 + (y - k)^2 = r^2$$

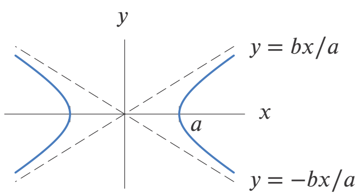
Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



Geometric Formulas

Conversion Between Radians and Degrees: π radians $= 180°$

Triangle

$A = \frac{1}{2}bh$

$= \frac{1}{2}ab\sin\theta$

Circle

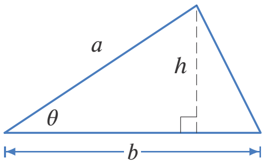
$A = \pi r^2$

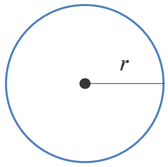
$C = 2\pi r$

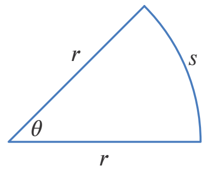
Sector of Circle

$A = \frac{1}{2}r^2\theta$ (θ in radians)

$s = r\theta$ (θ in radians)







Sphere

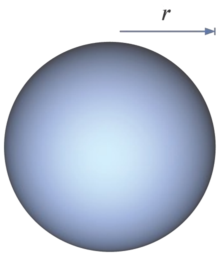
$V = \frac{4}{3}\pi r^3$ $A = 4\pi r^2$

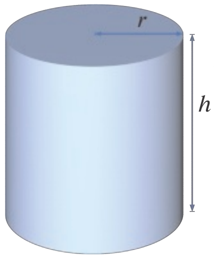
Cylinder

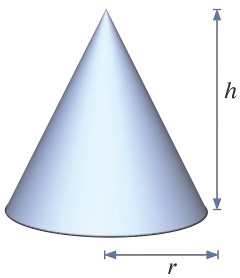
$V = \pi r^2 h$

Cone

$V = \frac{1}{3}\pi r^2 h$







Trigonometric Functions

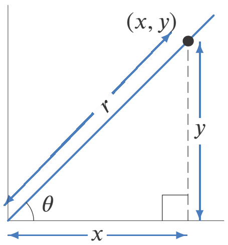
$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cos^2\theta + \sin^2\theta = 1$

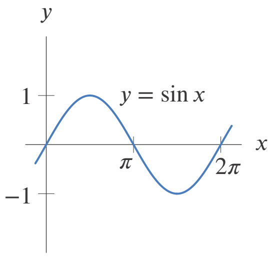


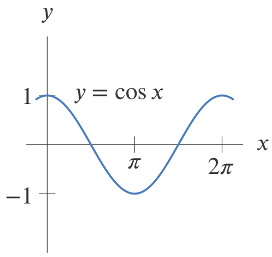
$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

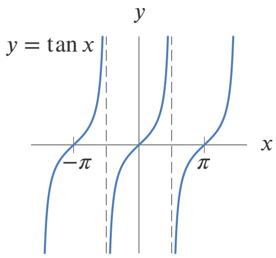
$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\sin(2A) = 2\sin A\cos A$

$\cos(2A) = 2\cos^2 A - 1 = 1 - 2\sin^2 A$







The Binomial Theorem

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{1\cdot 2}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^{n-3}y^3 + \cdots + nxy^{n-1} + y^n$$

$$(x-y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{1\cdot 2}x^{n-2}y^2 - \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^{n-3}y^3 + \cdots \pm nxy^{n-1} \mp y^n$$

CALCULUS

Seventh Edition

We dedicate this book to Andrew M. Gleason.

His brilliance and the extraordinary kindness and dignity with which he treated others made an enormous difference to us, and to many, many people. Andy brought out the best in everyone.

Deb Hughes Hallett
for the Calculus Consortium

CALCULUS

Seventh Edition

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PREFACE

Calculus is one of the greatest achievements of the human intellect. Inspired by problems in astronomy, Newton and Leibniz developed the ideas of calculus 300 years ago. Since then, each century has demonstrated the power of calculus to illuminate questions in mathematics, the physical sciences, engineering, and the social and biological sciences.

Calculus has been so successful both because its central theme—change—is pivotal to an analysis of the natural world and because of its extraordinary power to reduce complicated problems to simple procedures. Therein lies the danger in teaching calculus: it is possible to teach the subject as nothing but procedures—thereby losing sight of both the mathematics and of its practical value. This edition of *Calculus* continues our effort to promote courses in which understanding and computation reinforce each other. It reflects the input of users at research universities, four-year colleges, community colleges, and secondary schools, as well as of professionals in partner disciplines such as engineering and the natural and social sciences.

Mathematical Thinking Supported by Theory and Modeling

The first stage in the development of mathematical thinking is the acquisition of a clear intuitive picture of the central ideas. In the next stage, the student learns to reason with the intuitive ideas in plain English. After this foundation has been laid, there is a choice of direction. All students benefit from both theory and modeling, but the balance may differ for different groups. Some students, such as mathematics majors, may prefer more theory, while others may prefer more modeling. For instructors wishing to emphasize the connection between calculus and other fields, the text includes:

- A variety of problems from the **physical sciences** and **engineering**.
- Examples from the **biological sciences** and **economics**.
- Models from the **health sciences** and of **population growth**.
- Problems on **sustainability**.
- Case studies on **medicine** by David E. Sloane, MD.

Active Learning: Good Problems

As instructors ourselves, we know that interactive classrooms and well-crafted problems promote student learning. Since its inception, the hallmark of our text has been its innovative and engaging problems. These problems probe student understanding in ways often taken for granted. Praised for their creativity and variety, these problems have had influence far beyond the users of our textbook.

The Seventh Edition continues this tradition. Under our approach, which we call the "Rule of Four," ideas are presented graphically, numerically, symbolically, and verbally, thereby encouraging students to deepen their understanding. Graphs and tables in this text are assumed to show all necessary information about the functions they represent, including direction of change, local extrema, and discontinuities.

Problems in this text include:

- **Strengthen Your Understanding** problems at the end of every section. These problems ask students to reflect on what they have learned by deciding "What is wrong?" with a statement and to "Give an example" of an idea.
- **ConcepTests** promote active learning in the classroom. These can be used with or without personal response systems (*e.g.*, clickers), and have been shown to dramatically improve student learning. Available in a book or on the web at www.wiley.com/college/hughes-hallett.
- **Class Worksheets** allow instructors to engage students in individual or group class-work. Samples are available in the Instructor's Manual, and all are on the web at www.wiley.com/college/hughes-hallett.
- **Data and Models** Many examples and problems throughout the text involve data-driven models. For example, Section 11.7 has a series of problems studying the spread of the chikungunya virus that arrived

in the US in 2013. Projects at the end of each chapter of the E-Text (at www.wiley.com/college/hughes-hallett) provide opportunities for sustained investigation of real-world situations that can be modeled using calculus.

- **Drill Exercises** build student skill and confidence.

Enhancing Learning Online

This Seventh Edition provides opportunities for students to experience the concepts of calculus in ways that would not be possible in a traditional textbook. The E-Text of *Calculus*, powered by VitalSource, provides interactive demonstrations of concepts, embedded videos that illustrate problem-solving techniques, and built-in assessments that allow students to check their understanding as they read. The E-Text also contains additional content not found in the print edition:

- Worked example **videos** by Donna Krawczyk at the University of Arizona, which provide students the opportunity to see and hear hundreds of the book's examples being explained and worked out in detail
- Embedded **Interactive Explorations**, applets that present and explore key ideas graphically and dynamically—especially useful for display of three-dimensional graphs
- Material that reviews and extends the major ideas of each chapter: Chapter Summary, Review Exercises and Problems, CAS Challenge Problems, and Projects
- Challenging problems that involve further exploration and application of the mathematics in many sections
- Section on the ϵ, δ definition of limit (1.10)
- Appendices that include preliminary ideas useful in this course

Problems Available in WileyPLUS

Students and instructors can access a wide variety of problems through WileyPLUS with ORION, Wiley's digital learning environment. ORION Learning provides an adaptive, personalized learning experience that delivers easy-to-use analytics so instructors and students can see exactly where they're excelling and where they need help. WileyPLUS with ORION features the following resources:

- Online version of the text, featuring hyperlinks to referenced content, applets, videos, and supplements.
- Homework management tools, which enable the instructor to assign questions easily and grade them automatically, using a rich set of options and controls.
- QuickStart pre-designed reading and homework assignments. Use them as-is or customize them to fit the needs of your classroom.
- Intelligent Tutoring questions, in which students are prompted for responses as they step through a problem solution and receive targeted feedback based on those responses.
- Algebra & Trigonometry Refresher material, delivered through ORION, Wiley's personalized, adaptive learning environment that assesses students' readiness and provides students with an opportunity to brush up on material necessary to master Calculus, as well as to determine areas that require further review.

Online resources and support are also available through WebAssign. WebAssign for Hughes-Hallett Calculus Seventh Edition contains a vast range of assignable and autogradable homework questions as well as an Enhanced VitalSouce e-text with embedded videos, interatives, and questions.

Flexibility and Adaptability: Varied Approaches

The Seventh Edition of *Calculus* is designed to provide flexibility for instructors who have a range of preferences regarding inclusion of topics and applications and the use of computational technology. For those who prefer the lean topic list of earlier editions, we have kept clear the main conceptual paths. For example,

- The Key Concept chapters on the derivative and the definite integral (Chapters 2 and 5) can be covered at the outset of the course, right after Chapter 1.

- Limits and continuity (Sections 1.7, 1.8, and 1.9) can be covered in depth before the introduction of the derivative (Sections 2.1 and 2.2), or after.
- Approximating Functions Using Series (Chapter 10) can be covered before, or without, Chapter 9.
- In Chapter 4 (Using the Derivative), instructors can select freely from Sections 4.3–4.8.
- Chapter 8 (Using the Definite Integral) contains a wide range of applications. Instructors can select one or two to do in detail.

To use calculus effectively, students need skill in both symbolic manipulation and the use of technology. The balance between the two may vary, depending on the needs of the students and the wishes of the instructor. The book is adaptable to many different combinations.

The book does not require any specific software or technology. It has been used with graphing calculators, graphing software, and computer algebra systems. Any technology with the ability to graph functions and perform numerical integration will suffice. Students are expected to use their own judgment to determine where technology is useful.

Content

This content represents our vision of how calculus can be taught. It is flexible enough to accommodate individual course needs and requirements. Topics can easily be added or deleted, or the order changed.

Changes to the text in the Seventh Edition are in italics. In all chapters, problems were added and others were updated. In total, there are more than 900 new problems.

Chapter 1: A Library of Functions

This chapter introduces all the elementary functions to be used in the book. Although the functions are probably familiar, the graphical, numerical, verbal, and modeling approach to them may be new. We introduce exponential functions at the earliest possible stage, since they are fundamental to the understanding of real-world processes.

The content on limits and continuity in this chapter has been revised and expanded to emphasize the limit as a central idea of calculus. Section 1.7 gives an intuitive introduction to the ideas of limit and continuity. Section 1.8 introduces one-sided limits and limits at infinity and presents properties of limits of combinations of functions, such as sums and products. The new Section 1.9 gives a variety of algebraic techniques for computing limits, together with many new exercises and problems applying those techniques, and introduces the Squeeze Theorem. The new online Section 1.10 contains the ϵ, δ definition of limit, previously in Section 1.8.

Chapter 2: Key Concept: The Derivative

The purpose of this chapter is to give the student a practical understanding of the definition of the derivative and its interpretation as an instantaneous rate of change. The power rule is introduced; other rules are introduced in Chapter 3.

Chapter 3: Short-Cuts to Differentiation

The derivatives of all the functions in Chapter 1 are introduced, as well as the rules for differentiating products; quotients; and composite, inverse, hyperbolic, and implicitly defined functions.

Chapter 4: Using the Derivative

The aim of this chapter is to enable the student to use the derivative in solving problems, including optimization, graphing, rates, parametric equations, and indeterminate forms. It is not necessary to cover all the sections in this chapter.

Chapter 5: Key Concept: The Definite Integral

The purpose of this chapter is to give the student a practical understanding of the definite integral as a limit of Riemann sums and to bring out the connection between the derivative and the definite integral in the Fundamental Theorem of Calculus.

The difference between total distance traveled during a time interval is contrasted with the change in position.

Chapter 6: Constructing Antiderivatives

This chapter focuses on going backward from a derivative to the original function, first graphically and numerically, then analytically. It introduces the Second Fundamental Theorem of Calculus and the concept of a differential equation.

Chapter 7: Integration

This chapter includes several techniques of integration, including substitution, parts, partial fractions, and trigonometric substitutions; others are included in the table of integrals. There are discussions of numerical methods and of improper integrals.

Chapter 8: Using the Definite Integral

This chapter emphasizes the idea of subdividing a quantity to produce Riemann sums which, in the limit, yield a definite integral. It shows how the integral is used in geometry, physics, economics, and probability; polar coordinates are introduced. It is not necessary to cover all the sections in this chapter.

Distance traveled along a parametrically defined curve during a time interval is contrasted with arc length.

Chapter 9: Sequences and Series

This chapter focuses on sequences, series of constants, and convergence. It includes the integral, ratio, comparison, limit comparison, and alternating series tests. It also introduces geometric series and general power series, including their intervals of convergence.

Rearrangement of the terms of a conditionally convergent series is discussed.

Chapter 10: Approximating Functions

This chapter introduces Taylor Series and Fourier Series using the idea of approximating functions by simpler functions.

The term Maclaurin series is introduced for a Taylor series centered at 0. *Term-by-term differentiation of a Taylor series within its interval of convergence is introduced without proof. This term-by-term differentiation allows us to show that a power series is its own Taylor series.*

Chapter 11: Differential Equations

This chapter introduces differential equations. The emphasis is on qualitative solutions, modeling, and interpretation.

Appendices

There are online appendices on roots, accuracy, and bounds; complex numbers; Newton's method; and vectors in the plane. The appendix on vectors can be covered at any time, but may be particularly useful in the conjunction with Section 4.8 on parametric equations.

Supplementary Materials and Additional Resources

Supplements for the instructor can be obtained online at the book companion site or by contacting your Wiley representative. The following supplementary materials are available for this edition:

- **Instructor's Manual** containing teaching tips, calculator programs, overhead transparency masters, sample worksheets, and sample syllabi.
- **Computerized Test Bank**, comprised of nearly 7,000 questions, mostly algorithmically-generated, which allows for multiple versions of a single test or quiz.
- **Instructor's Solution Manual** with complete solutions to all problems.
- **Student Solution Manual** with complete solutions to half the odd-numbered problems.
- **Graphing Calculator Manual**, to help students get the most out of their graphing calculators, and to show how they can apply the numerical and graphing functions of their calculators to their study of calculus.
- **Additional Material**, elaborating specially marked points in the text and password-protected electronic versions of the instructor ancillaries, can be found on the web at www.wiley.com/college/hughes-hallett.

ConcepTests

ConcepTests, modeled on the pioneering work of Harvard physicist Eric Mazur, are questions designed to promote active learning during class, particularly (but not exclusively) in large lectures. Our evaluation data show students taught with ConcepTests outperformed students taught by traditional lecture methods 73% versus 17% on conceptual questions, and 63% versus 54% on computational problems.

Advanced Placement (AP) Teacher's Guide

The AP Guide, written by a team of experienced AP teachers, provides tips, multiple-choice questions, and free-response questions that correlate to each chapter of the text. It also features a collection of labs designed to complement the teaching of key AP Calculus concepts.

New material has been added to reflect recent changes in the learning objectives for AB and BC Calculus, including extended coverage of limits, continuity, sequences, and series. Also new to this edition are grids that align multiple choice and free-response questions to the College Board's Enduring Understandings, Learning Objectives, and Essential Knowledge.

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Karen Rhea
Ayşe Şahin
Adam Spiegler
Jeff Tecosky-Feldman
Thomas W. Tucker
Aaron D. Wootton

To Students: How to Learn from this Book

- This book may be different from other math textbooks that you have used, so it may be helpful to know about some of the differences in advance. This book emphasizes at every stage the *meaning* (in practical, graphical or numerical terms) of the symbols you are using. There is much less emphasis on "plug-and-chug" and using formulas, and much more emphasis on the interpretation of these formulas than you may expect. You will often be asked to explain your ideas in words or to explain an answer using graphs.
- The book contains the main ideas of calculus in plain English. Your success in using this book will depend on your reading, questioning, and thinking hard about the ideas presented. Although you may not have done this with other books, you should plan on reading the text in detail, not just the worked examples.
- There are very few examples in the text that are exactly like the homework problems. This means that you can't just look at a homework problem and search for a similar–looking "worked out" example. Success with the homework will come by grappling with the ideas of calculus.
- Many of the problems that we have included in the book are open-ended. This means that there may be more than one approach and more than one solution, depending on your analysis. Many times, solving a problem relies on common-sense ideas that are not stated in the problem but which you will know from everyday life.
- Some problems in this book assume that you have access to a graphing calculator or computer. There are many situations where you may not be able to find an exact solution to a problem, but you can use a calculator or computer to get a reasonable approximation.
- This book attempts to give equal weight to four methods for describing functions: graphical (a picture), numerical (a table of values), algebraic (a formula), and verbal. Sometimes you may find it easier to translate a problem given in one form into another. The best idea is to be flexible about your approach: if one way of looking at a problem doesn't work, try another.
- Students using this book have found discussing these problems in small groups very helpful. There are a great many problems which are not cut-and-dried; it can help to attack them with the other perspectives your col-

leagues can provide. If group work is not feasible, see if your instructor can organize a discussion session in which additional problems can be worked on.

- You are probably wondering what you'll get from the book. The answer is, if you put in a solid effort, you will get a real understanding of one of the most important accomplishments of the last millennium—calculus—as well as a real sense of the power of mathematics in the age of technology.

CONTENTS

1 FOUNDATION FOR CALCULUS: FUNCTIONS AND LIMITS 1

1.1 FUNCTIONS AND CHANGE 2
1.2 EXPONENTIAL FUNCTIONS 13
1.3 NEW FUNCTIONS FROM OLD 23
1.4 LOGARITHMIC FUNCTIONS 32
1.5 TRIGONOMETRIC FUNCTIONS 39
1.6 POWERS, POLYNOMIALS, AND RATIONAL FUNCTIONS 49
1.7 INTRODUCTION TO LIMITS AND CONTINUITY 58
1.8 EXTENDING THE IDEA OF A LIMIT 67
1.9 FURTHER LIMIT CALCULATIONS USING ALGEBRA 75
1.10 OPTIONAL PREVIEW OF THE FORMAL DEFINITION OF A LIMIT *ONLINE*
REVIEW PROBLEMS *ONLINE*
PROJECTS *ONLINE*

2 KEY CONCEPT: THE DERIVATIVE 83

2.1 HOW DO WE MEASURE SPEED? 84
2.2 THE DERIVATIVE AT A POINT 91
2.3 THE DERIVATIVE FUNCTION 99
2.4 INTERPRETATIONS OF THE DERIVATIVE 108
2.5 THE SECOND DERIVATIVE 115
2.6 DIFFERENTIABILITY 123
REVIEW PROBLEMS *ONLINE*
PROJECTS *ONLINE*

3 SHORT-CUTS TO DIFFERENTIATION 129

3.1 POWERS AND POLYNOMIALS 130
3.2 THE EXPONENTIAL FUNCTION 140
3.3 THE PRODUCT AND QUOTIENT RULES 144

For online material, see www.wiley.com/college/hughes-hallett.

3.4 THE CHAIN RULE 151
3.5 THE TRIGONOMETRIC FUNCTIONS 158
3.6 THE CHAIN RULE AND INVERSE FUNCTIONS 164
3.7 IMPLICIT FUNCTIONS 171
3.8 HYPERBOLIC FUNCTIONS 174
3.9 LINEAR APPROXIMATION AND THE DERIVATIVE 178
3.10 THEOREMS ABOUT DIFFERENTIABLE FUNCTIONS 186
REVIEW PROBLEMS *ONLINE*
PROJECTS *ONLINE*

4 USING THE DERIVATIVE 191

4.1 USING FIRST AND SECOND DERIVATIVES 192
4.2 OPTIMIZATION 203
4.3 OPTIMIZATION AND MODELING 212
4.4 FAMILIES OF FUNCTIONS AND MODELING 224
4.5 APPLICATIONS TO MARGINALITY 233
4.6 RATES AND RELATED RATES 243
4.7 L'HOPITAL'S RULE, GROWTH, AND DOMINANCE 252
4.8 PARAMETRIC EQUATIONS 259
REVIEW PROBLEMS *ONLINE*
PROJECTS *ONLINE*

5 KEY CONCEPT: THE DEFINITE INTEGRAL 271

5.1 HOW DO WE MEASURE DISTANCE TRAVELED? 272
5.2 THE DEFINITE INTEGRAL 283
5.3 THE FUNDAMENTAL THEOREM AND INTERPRETATIONS 292
5.4 THEOREMS ABOUT DEFINITE INTEGRALS 302
REVIEW PROBLEMS *ONLINE*
PROJECTS *ONLINE*

6 CONSTRUCTING ANTIDERIVATIVES 315

6.1 ANTIDERIVATIVES GRAPHICALLY AND NUMERICALLY 316
6.2 CONSTRUCTING ANTIDERIVATIVES ANALYTICALLY 322

6.3 DIFFERENTIAL EQUATIONS AND MOTION 329
6.4 SECOND FUNDAMENTAL THEOREM OF CALCULUS 335
REVIEW PROBLEMS *ONLINE*
PROJECTS *ONLINE*

7 INTEGRATION 341

7.1 INTEGRATION BY SUBSTITUTION 342
7.2 INTEGRATION BY PARTS 353
7.3 TABLES OF INTEGRALS 360
7.4 ALGEBRAIC IDENTITIES AND TRIGONOMETRIC SUBSTITUTIONS 366
7.5 NUMERICAL METHODS FOR DEFINITE INTEGRALS 376
7.6 IMPROPER INTEGRALS 385
7.7 COMPARISON OF IMPROPER INTEGRALS 394
REVIEW PROBLEMS *ONLINE*
PROJECTS *ONLINE*

8 USING THE DEFINITE INTEGRAL 401

8.1 AREAS AND VOLUMES 402
8.2 APPLICATIONS TO GEOMETRY 410
8.3 AREA AND ARC LENGTH IN POLAR COORDINATES 420
8.4 DENSITY AND CENTER OF MASS 429
8.5 APPLICATIONS TO PHYSICS 439
8.6 APPLICATIONS TO ECONOMICS 450
8.7 DISTRIBUTION FUNCTIONS 457
8.8 PROBABILITY, MEAN, AND MEDIAN 464
REVIEW PROBLEMS *ONLINE*
PROJECTS *ONLINE*

9 SEQUENCES AND SERIES 473

9.1 SEQUENCES 474
9.2 GEOMETRIC SERIES 480
9.3 CONVERGENCE OF SERIES 488
9.4 TESTS FOR CONVERGENCE 494

9.5 POWER SERIES AND INTERVAL OF CONVERGENCE 504
REVIEW PROBLEMS *ONLINE*
PROJECTS *ONLINE*

10 APPROXIMATING FUNCTIONS USING SERIES 513

10.1 TAYLOR POLYNOMIALS 514
10.2 TAYLOR SERIES 523
10.3 FINDING AND USING TAYLOR SERIES 530
10.4 THE ERROR IN TAYLOR POLYNOMIAL APPROXIMATIONS 539
10.5 FOURIER SERIES 546
REVIEW PROBLEMS *ONLINE*
PROJECTS *ONLINE*

11 DIFFERENTIAL EQUATIONS 561

11.1 WHAT IS A DIFFERENTIAL EQUATION? 562
11.2 SLOPE FIELDS 567
11.3 EULER'S METHOD 575
11.4 SEPARATION OF VARIABLES 580
11.5 GROWTH AND DECAY 586
11.6 APPLICATIONS AND MODELING 597
11.7 THE LOGISTIC MODEL 606
11.8 SYSTEMS OF DIFFERENTIAL EQUATIONS 616
11.9 ANALYZING THE PHASE PLANE 626
11.10 SECOND-ORDER DIFFERENTIAL EQUATIONS: OSCILLATIONS 632
11.11 LINEAR SECOND-ORDER DIFFERENTIAL EQUATIONS 640
REVIEW PROBLEMS *ONLINE*
PROJECTS *ONLINE*

APPENDICES *Online*

A ROOTS, ACCURACY, AND BOUNDS *ONLINE*
B COMPLEX NUMBERS *ONLINE*
C NEWTON'S METHOD *ONLINE*
D VECTORS IN THE PLANE *ONLINE*

READY REFERENCE 651

ANSWERS TO ODD-NUMBERED PROBLEMS 663

INDEX 707

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Chapter One

FOUNDATION FOR CALCULUS: FUNCTIONS AND LIMITS

Contents

1.1 Functions and Change 2
- The Rule of Four 2
- Examples of Domain and Range 3
- Linear Functions 4
- Families of Linear Functions 5
- Increasing versus Decreasing Functions 6
- Proportionality 6

1.2 Exponential Functions 13
- Concavity 14
- Elimination of a Drug from the Body 15
- The General Exponential Function 15
- Half-Life and Doubling Time 16
- The Family of Exponential Functions 16
- Exponential Functions with Base *e* 17

1.3 New Functions from Old 23
- Shifts and Stretches 23
- Composite Functions 24
- Odd and Even Functions: Symmetry 25
- Inverse Functions 26

1.4 Logarithmic Functions 32
- Logarithms to Base 10 and to Base *e* 32
- Solving Equations Using Logarithms 33

1.5 Trigonometric Functions 39
- Radians 39
- The Sine and Cosine Functions 40
- The Tangent Function 43
- The Inverse Trigonometric Functions 44

1.6 Powers, Polynomials, and Rational Functions . . . 49
- Power Functions 49
- Dominance 50
- Polynomials 51
- Rational Functions 53

1.7 Introduction to Limits and Continuity 58
- The Idea of Continuity 58
- The Idea of a Limit 59
- Definition of Limit 60
- Definition of Continuity 60
  - The Intermediate Value Theorem 60
- Finding Limits Exactly Using Continuity and Algebra 61

1.8 Extending the Idea of a Limit 67
- One-Sided Limits 67
- Limits and Asymptotes 68

1.9 Further Limit Calculations using Algebra 75
- Limits of Quotients 75
- Calculating Limits at Infinity 78
- The Squeeze Theorem 79

1.1 FUNCTIONS AND CHANGE

In mathematics, a *function* is used to represent the dependence of one quantity upon another.

Let's look at an example. In 2015, Boston, Massachusetts, had the highest annual snowfall, 110.6 inches, since recording started in 1872. Table 1.1 shows one 14-day period in which the city broke another record with a total of 64.4 inches.[1]

Table 1.1 *Daily snowfall in inches for Boston, January 27 to February 9, 2015*

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Snowfall	22.1	0.2	0	0.7	1.3	0	16.2	0	0	0.8	0	0.9	7.4	14.8

You may not have thought of something so unpredictable as daily snowfall as being a function, but it *is* a function of day, because each day gives rise to one snowfall total. There is no formula for the daily snowfall (otherwise we would not need a weather bureau), but nevertheless the daily snowfall in Boston does satisfy the definition of a function: Each day, t, has a unique snowfall, S, associated with it.

We define a function as follows:

> A **function** is a rule that takes certain numbers as inputs and assigns to each a definite output number. The set of all input numbers is called the **domain** of the function and the set of resulting output numbers is called the **range** of the function.

The input is called the *independent variable* and the output is called the *dependent variable*. In the snowfall example, the domain is the set of days $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$ and the range is the set of daily snowfalls $\{0, 0.2, 0.7, 0.8, 0.9, 1.3, 7.4, 14.8, 16.2, 22.1\}$. We call the function f and write $S = f(t)$. Notice that a function may have identical outputs for different inputs (Days 8 and 9, for example).

Some quantities, such as a day or date, are *discrete*, meaning they take only certain isolated values (days must be integers). Other quantities, such as time, are *continuous* as they can be any number. For a continuous variable, domains and ranges are often written using interval notation:

The set of numbers t such that $a \le t \le b$ is called a *closed interval* and written $[a, b]$.

The set of numbers t such that $a < t < b$ is called an *open interval* and written (a, b).

The Rule of Four: Tables, Graphs, Formulas, and Words

Functions can be represented by tables, graphs, formulas, and descriptions in words. For example, the function giving the daily snowfall in Boston can be represented by the graph in Figure 1.1, as well as by Table 1.1.

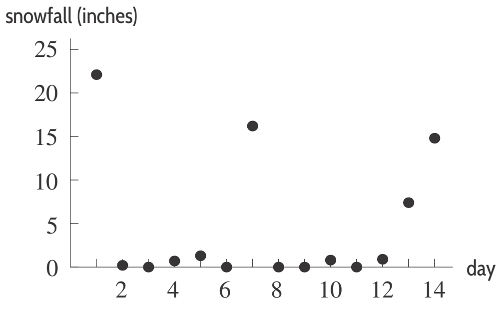


Figure 1.1: Boston snowfall, starting January 27, 2015

As another example of a function, consider the snowy tree cricket. Surprisingly enough, all such crickets chirp at essentially the same rate if they are at the same temperature. That means that the chirp rate is a function of temperature. In other words, if we know the temperature, we can determine

[1]http://w2.weather.gov/climate/xmacis.php?wfo=box. Accessed June 2015.

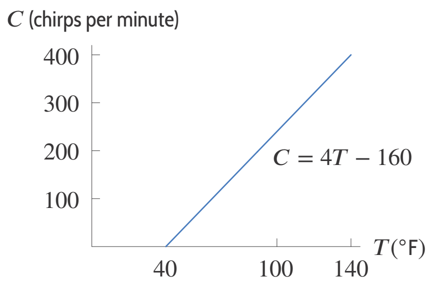


Figure 1.2: Cricket chirp rate versus temperature

the chirp rate. Even more surprisingly, the chirp rate, C, in chirps per minute, increases steadily with the temperature, T, in degrees Fahrenheit, and can be computed by the formula

$$C = 4T - 160$$

to a fair level of accuracy. We write $C = f(T)$ to express the fact that we think of C as a function of T and that we have named this function f. The graph of this function is in Figure 1.2.

Notice that the graph of $C = f(T)$ in Figure 1.2 is a solid line. This is because $C = f(T)$ is a *continuous function*. Roughly speaking, a continuous function is one whose graph has no breaks, jumps, or holes. This means that the independent variable must be continuous. (We give a more precise definition of continuity of a function in Section 1.7.)

Examples of Domain and Range

If the domain of a function is not specified, we usually take it to be the largest possible set of real numbers. For example, we usually think of the domain of the function $f(x) = x^2$ as all real numbers. However, the domain of the function $g(x) = 1/x$ is all real numbers except zero, since we cannot divide by zero.

Sometimes we restrict the domain to be smaller than the largest possible set of real numbers. For example, if the function $f(x) = x^2$ is used to represent the area of a square of side x, we restrict the domain to nonnegative values of x.

Example 1 The function $C = f(T)$ gives chirp rate as a function of temperature. We restrict this function to temperatures for which the predicted chirp rate is positive, and up to the highest temperature ever recorded at a weather station, 134°F. What is the domain of this function f?

Solution If we consider the equation

$$C = 4T - 160$$

simply as a mathematical relationship between two variables C and T, any T value is possible. However, if we think of it as a relationship between cricket chirps and temperature, then C cannot be less than 0. Since $C = 0$ leads to $0 = 4T - 160$, and so $T = 40$°F, we see that T cannot be less than 40°F. (See Figure 1.2.) In addition, we are told that the function is not defined for temperatures above 134°. Thus, for the function $C = f(T)$ we have

$$\begin{aligned} \text{Domain} &= \text{All } T \text{ values between 40°F and 134°F} \\ &= \text{All } T \text{ values with } 40 \le T \le 134 \\ &= [40, 134]. \end{aligned}$$

Example 2 Find the range of the function f, given the domain from Example 1. In other words, find all possible values of the chirp rate, C, in the equation $C = f(T)$.

Solution Again, if we consider $C = 4T - 160$ simply as a mathematical relationship, its range is all real C values. However, when thinking of the meaning of $C = f(T)$ for crickets, we see that the function predicts cricket chirps per minute between 0 (at $T = 40$°F) and 376 (at $T = 134$°F). Hence,

$$\begin{aligned} \text{Range} &= \text{All } C \text{ values from 0 to 376} \\ &= \text{All } C \text{ values with } 0 \le C \le 376 \\ &= [0, 376]. \end{aligned}$$

In using the temperature to predict the chirp rate, we thought of the temperature as the *independent variable* and the chirp rate as the *dependent variable*. However, we could do this backward, and calculate the temperature from the chirp rate. From this point of view, the temperature is dependent on the chirp rate. Thus, which variable is dependent and which is independent may depend on your viewpoint.

Linear Functions

The chirp-rate function, $C = f(T)$, is an example of a *linear function*. A function is linear if its slope, or rate of change, is the same at every point. The rate of change of a function that is not linear may vary from point to point.

Olympic and World Records

During the early years of the Olympics, the height of the men's winning pole vault increased approximately 8 inches every four years. Table 1.2 shows that the height started at 130 inches in 1900, and increased by the equivalent of 2 inches a year. So the height was a linear function of time from 1900 to 1912. If y is the winning height in inches and t is the number of years since 1900, we can write

$$y = f(t) = 130 + 2t.$$

Since $y = f(t)$ increases with t, we say that f is an *increasing function*. The coefficient 2 tells us the rate, in inches per year, at which the height increases.

Table 1.2 *Men's Olympic pole vault winning height (approximate)*

Year	1900	1904	1908	1912
Height (inches)	130	138	146	154

This rate of increase is the *slope* of the line in Figure 1.3. The slope is given by the ratio

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{146 - 138}{8 - 4} = \frac{8}{4} = 2 \text{ inches/year}.$$

Calculating the slope (rise/run) using any other two points on the line gives the same value.

What about the constant 130? This represents the initial height in 1900, when $t = 0$. Geometrically, 130 is the *intercept* on the vertical axis.

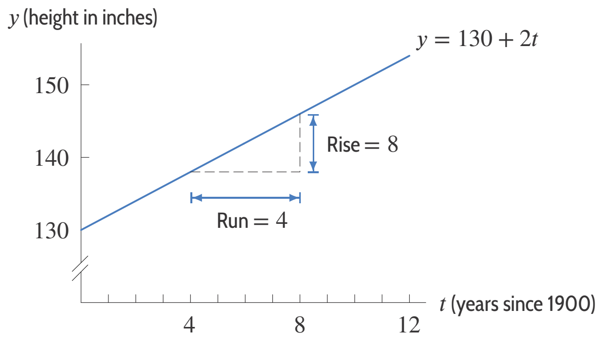


Figure 1.3: Olympic pole vault records

You may wonder whether the linear trend continues beyond 1912. Not surprisingly, it does not exactly. The formula $y = 130 + 2t$ predicts that the height in the 2012 Olympics would be 354 inches or 29 feet 6 inches, which is considerably higher than the actual value of 19 feet 7.05 inches. There is clearly a danger in *extrapolating* too far from the given data. You should also observe that the data in Table 1.2 is discrete, because it is given only at specific points (every four years). However, we have treated the variable t as though it were continuous, because the function $y = 130 + 2t$ makes sense for all values of t. The graph in Figure 1.3 is of the continuous function because it is a solid line, rather than four separate points representing the years in which the Olympics were held.

As the pole vault heights have increased over the years, the time to run the mile has decreased. If y is the world record time to run the mile, in seconds, and t is the number of years since 1900, then records show that, approximately,

$$y = g(t) = 260 - 0.39t.$$

The 260 tells us that the world record was 260 seconds in 1900 (at $t = 0$). The slope, −0.39, tells us that the world record decreased by about 0.39 seconds per year. We say that g is a *decreasing function*.

Difference Quotients and Delta Notation

We use the symbol Δ (the Greek letter capital delta) to mean "change in," so Δx means change in x and Δy means change in y.

The slope of a linear function $y = f(x)$ can be calculated from values of the function at two points, given by x_1 and x_2, using the formula

$$m = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

The quantity $(f(x_2) - f(x_1))/(x_2 - x_1)$ is called a *difference quotient* because it is the quotient of two differences. (See Figure 1.4.) Since $m = \Delta y/\Delta x$, the units of m are y-units over x-units.

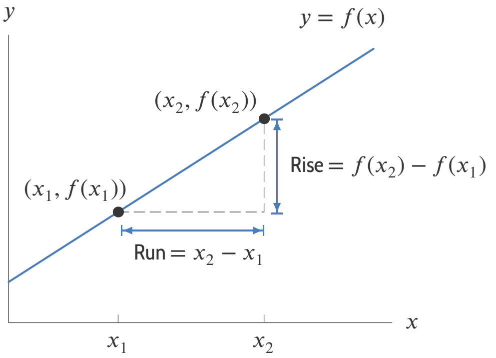


Figure 1.4: Difference quotient $= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

Families of Linear Functions

A **linear function** has the form

$$y = f(x) = b + mx.$$

Its graph is a line such that
- m is the **slope**, or rate of change of y with respect to x.
- b is the **vertical intercept**, or value of y when x is zero.

Notice that if the slope, m, is zero, we have $y = b$, a horizontal line.

To recognize that a table of x and y values comes from a linear function, $y = b + mx$, look for differences in y-values that are constant for equally spaced x-values.

Formulas such as $f(x) = b + mx$, in which the constants m and b can take on various values, give a *family of functions*. All the functions in a family share certain properties—in this case, all the graphs are straight lines. The constants m and b are called *parameters*; their meaning is shown in Figures 1.5 and 1.6. Notice that the greater the magnitude of m, the steeper the line.

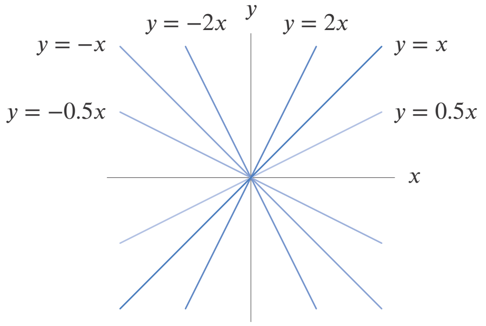


Figure 1.5: The family $y = mx$ (with $b = 0$)

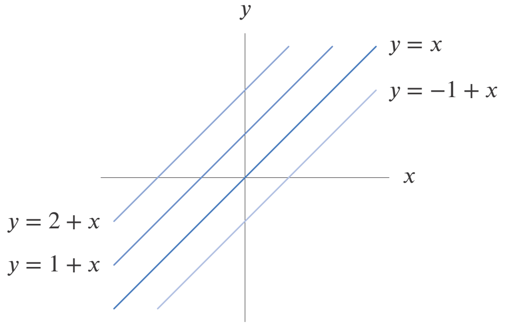


Figure 1.6: The family $y = b + x$ (with $m = 1$)

Increasing versus Decreasing Functions

The terms increasing and decreasing can be applied to other functions, not just linear ones. See Figure 1.7. In general,

> A function f is **increasing** if the values of $f(x)$ increase as x increases.
> A function f is **decreasing** if the values of $f(x)$ decrease as x increases.
>
> The graph of an *increasing* function *climbs* as we move from left to right.
> The graph of a *decreasing* function *falls* as we move from left to right.
>
> A function $f(x)$ is **monotonic** if it increases for all x or decreases for all x.

Figure 1.7: Increasing and decreasing functions

Proportionality

A common functional relationship occurs when one quantity is *proportional* to another. For example, the area, A, of a circle is proportional to the square of the radius, r, because

$$A = f(r) = \pi r^2.$$

> We say y is (directly) **proportional** to x if there is a nonzero constant k such that
>
> $$y = kx.$$
>
> This k is called the constant of proportionality.

We also say that one quantity is *inversely proportional* to another if one is proportional to the reciprocal of the other. For example, the speed, v, at which you make a 50-mile trip is inversely proportional to the time, t, taken, because v is proportional to $1/t$:

$$v = 50\left(\frac{1}{t}\right) = \frac{50}{t}.$$

Exercises and Problems for Section 1.1

EXERCISES

1. The population of a city, P, in millions, is a function of t, the number of years since 2010, so $P = f(t)$. Explain the meaning of the statement $f(5) = 7$ in terms of the population of this city.

2. The pollutant PCB (polychlorinated biphenyl) can affect the thickness of pelican eggshells. Thinking of the thickness, T, of the eggshells, in mm, as a function of the concentration, P, of PCBs in ppm (parts per million), we have $T = f(P)$. Explain the meaning of $f(200)$ in terms of thickness of pelican eggs and concentration of PCBs.

3. Describe what Figure 1.8 tells you about an assembly line whose productivity is represented as a function of the number of workers on the line.

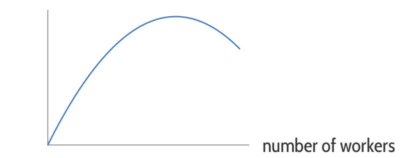


Figure 1.8

■ For Exercises 4–7, find an equation for the line that passes through the given points.

4. (0, 0) and (1, 1)
5. (0, 2) and (2, 3)
6. (−2, 1) and (2, 3)
7. (−1, 0) and (2, 6)

■ For Exercises 8–11, determine the slope and the y-intercept of the line whose equation is given.

8. $2y + 5x - 8 = 0$
9. $7y + 12x - 2 = 0$
10. $-4y + 2x + 8 = 0$
11. $12x = 6y + 4$

12. Match the graphs in Figure 1.9 with the following equations. (Note that the x and y scales may be unequal.)

(a) $y = x - 5$ **(b)** $-3x + 4 = y$
(c) $5 = y$ **(d)** $y = -4x - 5$
(e) $y = x + 6$ **(f)** $y = x/2$

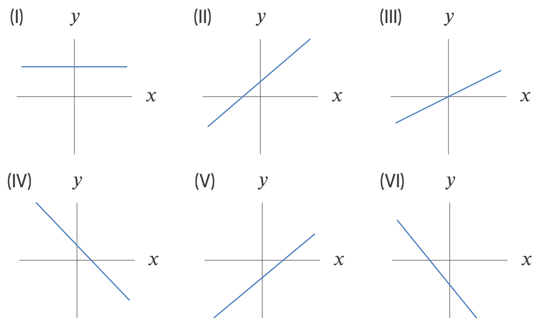


Figure 1.9

13. Match the graphs in Figure 1.10 with the following equations. (Note that the x and y scales may be unequal.)

(a) $y = -2.72x$ **(b)** $y = 0.01 + 0.001x$
(c) $y = 27.9 - 0.1x$ **(d)** $y = 0.1x - 27.9$
(e) $y = -5.7 - 200x$ **(f)** $y = x/3.14$

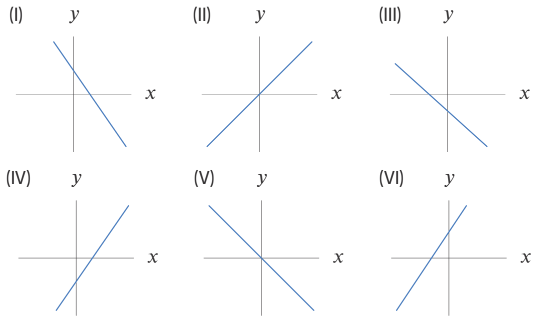


Figure 1.10

14. Estimate the slope and the equation of the line in Figure 1.11.

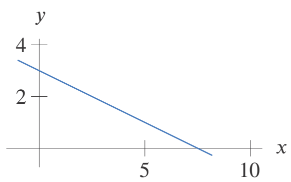


Figure 1.11

15. Find an equation for the line with slope m through the point (a, c).

16. Find a linear function that generates the values in Table 1.3.

Table 1.3

x	5.2	5.3	5.4	5.5	5.6
y	27.8	29.2	30.6	32.0	33.4

■ For Exercises 17–19, use the facts that parallel lines have equal slopes and that the slopes of perpendicular lines are negative reciprocals of one another.

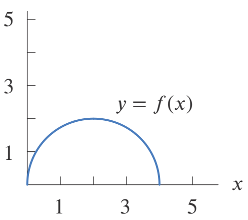
17. Find an equation for the line through the point (2, 1) which is perpendicular to the line $y = 5x - 3$.

18. Find equations for the lines through the point (1, 5) that are parallel to and perpendicular to the line with equation $y + 4x = 7$.

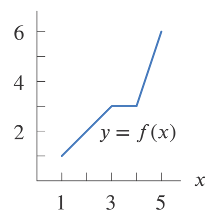
19. Find equations for the lines through the point (a, b) that are parallel and perpendicular to the line $y = mx + c$, assuming $m \neq 0$.

■ For Exercises 20–23, give the approximate domain and range of each function. Assume the entire graph is shown.

20.



21.



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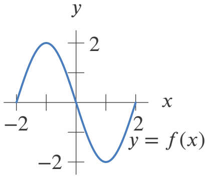
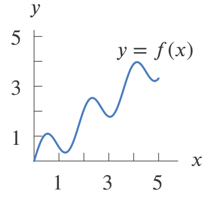


23.



■ Find the domain and range in Exercises 24–25.

24. $y = x^2 + 2$

25. $y = \dfrac{1}{x^2 + 2}$

26. If $f(t) = \sqrt{t^2 - 16}$, find all values of t for which $f(t)$ is a real number. Solve $f(t) = 3$.

■ In Exercises 27–31, write a formula representing the function.

27. The volume of a sphere is proportional to the cube of its radius, r.

28. The average velocity, v, for a trip over a fixed distance, d, is inversely proportional to the time of travel, t.

29. The strength, S, of a beam is proportional to the square of its thickness, h.

30. The energy, E, expended by a swimming dolphin is proportional to the cube of the speed, v, of the dolphin.

31. The number of animal species, N, of a certain body length, l, is inversely proportional to the square of l.

PROBLEMS

32. In December 2010, the snowfall in Minneapolis was unusually high,[2] leading to the collapse of the roof of the Metrodome. Figure 1.12 gives the snowfall, S, in Minneapolis for December 6–15, 2010.

(a) How do you know that the snowfall data represents a function of date?
(b) Estimate the snowfall on December 12.
(c) On which day was the snowfall more than 10 inches?
(d) During which consecutive two-day interval was the increase in snowfall largest?

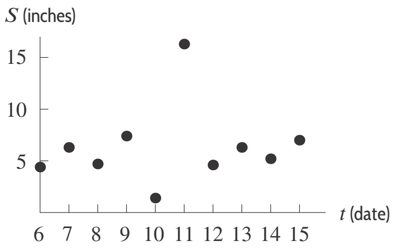


Figure 1.12

33. The value of a car, $V = f(a)$, in thousands of dollars, is a function of the age of the car, a, in years.

(a) Interpret the statement $f(5) = 6$.
(b) Sketch a possible graph of V against a. Is f an increasing or decreasing function? Explain.
(c) Explain the significance of the horizontal and vertical intercepts in terms of the value of the car.

34. Which graph in Figure 1.13 best matches each of the following stories?[3] Write a story for the remaining graph.

(a) I had just left home when I realized I had forgotten my books, so I went back to pick them up.
(b) Things went fine until I had a flat tire.
(c) I started out calmly but sped up when I realized I was going to be late.

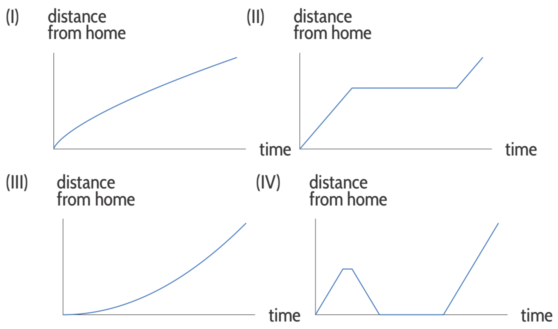


Figure 1.13

■ In Problems 35–38 the function $S = f(t)$ gives the average annual sea level, S, in meters, in Aberdeen, Scotland,[4]

[2] http://www.crh.noaa.gov/mpx/Climate/DisplayRecords.php
[3] Adapted from Jan Terwel, "Real Math in Cooperative Groups in Secondary Education." *Cooperative Learning in Mathematics*, ed. Neal Davidson, p. 234 (Reading: Addison Wesley, 1990).
[4] www.gov.uk, accessed January 7, 2015.

as a function of t, the number of years before 2012. Write a mathematical expression that represents the given statement.

35. In 2000 the average annual sea level in Aberdeen was 7.049 meters.

36. The average annual sea level in Aberdeen in 2012.

37. The average annual sea level in Aberdeen was the same in 1949 and 2000.

38. The average annual sea level in Aberdeen decreased by 8 millimeters from 2011 to 2012.

■ Problems 39–42 ask you to plot graphs based on the following story: "As I drove down the highway this morning, at first traffic was fast and uncongested, then it crept nearly bumper-to-bumper until we passed an accident, after which traffic flow went back to normal until I exited."

39. Driving speed against time on the highway

40. Distance driven against time on the highway

41. Distance from my exit vs time on the highway

42. Distance between cars vs distance driven on the highway

43. An object is put outside on a cold day at time $t = 0$. Its temperature, $H = f(t)$, in °C, is graphed in Figure 1.14.

- **(a)** What does the statement $f(30) = 10$ mean in terms of temperature? Include units for 30 and for 10 in your answer.
- **(b)** Explain what the vertical intercept, a, and the horizontal intercept, b, represent in terms of temperature of the object and time outside.

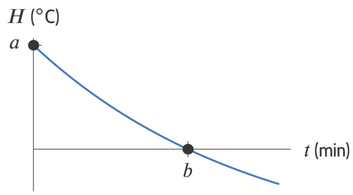


Figure 1.14

44. A rock is dropped from a window and falls to the ground below. The height, s (in meters), of the rock above ground is a function of the time, t (in seconds), since the rock was dropped, so $s = f(t)$.

- **(a)** Sketch a possible graph of s as a function of t.
- **(b)** Explain what the statement $f(7) = 12$ tells us about the rock's fall.
- **(c)** The graph drawn as the answer for part (a) should have a horizontal and vertical intercept. Interpret each intercept in terms of the rock's fall.

45. You drive at a constant speed from Chicago to Detroit, a distance of 275 miles. About 120 miles from Chicago you pass through Kalamazoo, Michigan. Sketch a graph of your distance from Kalamazoo as a function of time.

46. US imports of crude oil and petroleum have been increasing.[5] There have been many ups and downs, but the general trend is shown by the line in Figure 1.15.

- **(a)** Find the slope of the line. Include its units of measurement.
- **(b)** Write an equation for the line. Define your variables, including their units.
- **(c)** Assuming the trend continues, when does the linear model predict imports will reach 18 million barrels per day? Do you think this is a reliable prediction? Give reasons.

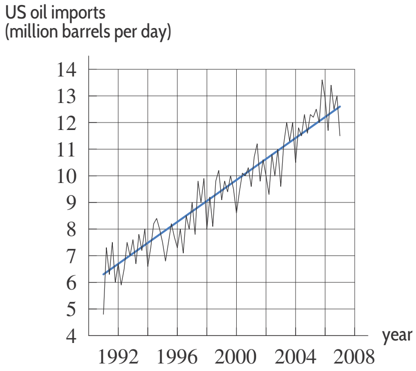


Figure 1.15

■ Problems 47–49 use Figure 1.16 showing how the quantity, Q, of grass (kg/hectare) in different parts of Namibia depended on the average annual rainfall, r, (mm), in two different years.[6]

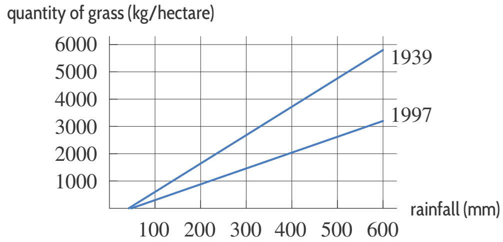


Figure 1.16

47. **(a)** For 1939, find the slope of the line, including units.
(b) Interpret the slope in this context.
(c) Find the equation of the line.

48. **(a)** For 1997, find the slope of the line, including units.
(b) Interpret the slope in this context.
(c) Find the equation of the line.

49. Which of the two functions in Figure 1.16 has the larger difference quotient $\Delta Q/\Delta r$? What does this tell us about grass in Namibia?

[5] http://www.theoildrum.com/node/2767. Accessed May 2015.

[6] David Ward and Ben T. Ngairorue, "Are Namibia's Grasslands Desertifying?", *Journal of Range Management* 53, 2000, 138–144.